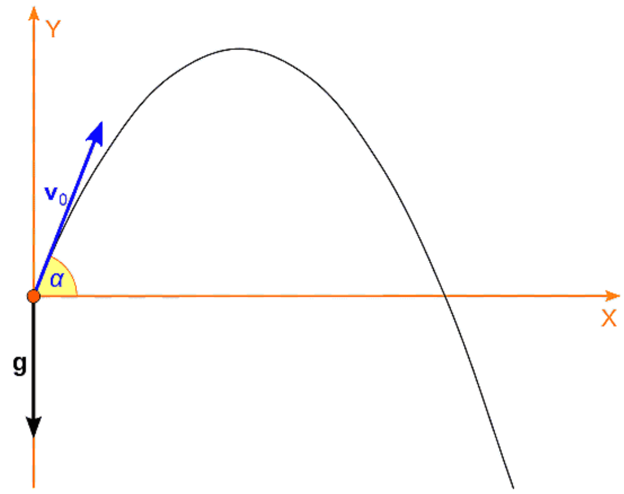
Constraint Programming

2021/2022 – Mini-Test #2

Monday, 3 January, 9:00 h in 127-Ed.II

Duration: 1.5 h (open book)

The motion of a projectile through the air that is subject only to the acceleration of gravity (g=9.81 ms^{-2}) is represented in the figure bellow.



Given the launch angle α (radians) and the initial velocity v_0 (ms^{-1}), the projectile follows a parabolic trajectory represented by the following equation:

$$y = (\tan\alpha)x - \frac{g}{2v_0^2(\cos\alpha)^2}x^2$$

1. Interval Arithmetic

The horizontal range r of the projectile is the horizontal distance it has traveled when it returns to its initial height (y=0). Given the initial velocity $v_0 = 10$, and solving the above equation with respect to x, the horizontal range r can be computed as a function of α:

$$r = f(\alpha) = \frac{2v_0^2}{g}(\sin\alpha)(\cos\alpha)$$

In the following questions assume that $v_0 = 10$ ms^{-1} and g=9.81 ms^{-2}.

1.1. Define $F_n(I)$ the natural interval extension of f.

1.2. Define $F_c(I)$ the mean value extension of f over the interval $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ centered at the midpoint.

Some derivative rules:

$$\big(f(x) \times g(x)\big)' = f(x) \times \big(g(x)\big)' + \big(f(x)\big)' \times g(x)$$
$$(\sin x)' = (\cos x)$$
$$(\cos x)' = -(\sin x)$$

1.3. Given that: $2(\sin\alpha)(\cos\alpha) = \sin 2\alpha$

Define $F_r(I)$ that computes the sharpest enclosure of the range of f for any $I \subseteq \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$.

1.4. Compute $F_n\left(\left[\frac{\pi}{6}, \frac{\pi}{3}\right]\right)$, $F_c\left(\left[\frac{\pi}{6}, \frac{\pi}{3}\right]\right)$ and $F_r\left(\left[\frac{\pi}{6}, \frac{\pi}{3}\right]\right)$.

1.5. From the results obtained in 1.4 what can you conclude about the truth value of the following propositions? (Justify your answers)

a. $\exists_{\frac{\pi}{6} \le \alpha \le \frac{\pi}{3}} f(\alpha) = 8$

b. $\exists_{\frac{\pi}{6} \le \alpha \le \frac{\pi}{3}} f(\alpha) = 10$

2. Interval Newton

Consider the function: $h(\alpha) = r - \frac{2v_0^2}{g}(\sin\alpha)(\cos\alpha)$

Notice that when $h(\alpha) = 0$, the projectile launched with an angle α radians and initial velocity v_0 ms^{-1}, reach the horizontal range r.

2.1. Define the interval Newton function with respect to h (do not assign values to v_0 and r).

2.2. Use the Newton function to prove that with an initial velocity of 10 ms^{-1} there is no angle α between $\frac{\pi}{6}$ and $\frac{\pi}{3}$ to obtain a trajectory range smaller than 5 m.

2.3. Use the Newton function to prove that with an initial velocity of 10 ms^{-1} there is an angle α between $\frac{\pi}{6}$ and $\frac{\pi}{5}$ to obtain a trajectory range of 9 m and compute an interval enclosure of such angle with width not larger than 0.02 radians.

3. Constraint Propagation

Consider the constraint below obtained from the trajectory equation with angle α = $\frac{\pi}{4}$ radians:

$$y = x - 9.81\left(\frac{x}{v_0}\right)^2$$

3.1. Is the constraint box-consistent in box $B[x, y, v_0]$ = [4,5]×[2,3]×[9,10]?

3.2. Is the constraint hull-consistent in box $B[x, y, v_0]$ = [4,5]×[2,3]×[9,10]?

3.3. Can you reduce box *B* by applying HC4-revise to the constraint? Justify.

3.4. Apply HC4-revise to the constraint with an initial box *B'*$[x, y, v_0]$ = [4]×[2,3]×[8,10].

3.5. What is the box obtained by applying BC3-revise to the constraint with the same initial box *B'*?