Constraint Programming

2021/2022 – Exam

Friday, 21 January 2022, 14:30 h

Part II – Continuous Domains (1.5 h – open book)

1. Interval Arithmetic

Consider the following univariate quadratic function which can be expressed in three equivalent forms:

- $f(x) = x^2 - 7x + 10$ (standard form)
- $f(x) = (x - 5)(x - 2)$ (factored form)
- $f(x) = \left(x - \frac{7}{2}\right)^2 - \frac{9}{4}$ (vertex form)

1.1. Compute the natural interval evaluation of each form for I=[4,6].

1.2. Prove that the natural interval evaluation of the factored form is exact for I=[a,b] with a≥5.

1.3. Define an algorithm that computes the exact bounds of the function for any I=[a,b].

2. Interval Newton

Consider the univariate quadratic function of the previous question.

2.1. Define the respective interval Newton function.

2.2. Use the interval Newton method to prove that there exists a unique root of $f(x)$ in [4.5,6.5].

2.3. Starting with [4.5,6.5] as the initial enclosure of the root, how many iterations of the interval Newton method would guarantee an enclosure width smaller than 10^{-8}. Justify.

3. Constraint Propagation

Consider the constraint $y \geq \left(x - \frac{7}{2}\right)^2 - \frac{9}{4}$ and the box $B[x, y]$ = [4.5, 5.5]×[-1,0].

3.1. Is the constraint box-consistent in box B?

3.2. Is the constraint hull-consistent in box B?

3.3. Apply HC4-revise to the constraint with the initial box B.