Interval Newton Method. Interval Linear Systems. (see Lecture3.pdf)

1. Nonlinear Equations

Apply the interval Newton method to compute enclosures of all the zeros of the following equations:

a) $f(x) = \sin(\sqrt{x}) - x$ with $x \in [0,1]$ b) $f(x) = -2.75x^3 + 18x^2 - 21x - 12$ with $x \in [-2,6]$ c) $f(x) = \sin(10x) + \cos(3x)$ with $x \in [3,6]$

2. Interval Linear Systems

Consider the interval linear system:

$$\begin{bmatrix} \frac{2}{5}, \frac{4}{5} \end{bmatrix} x_1 + x_2 = \begin{bmatrix} \frac{15}{5}, \frac{17}{5} \end{bmatrix}$$
$$\begin{bmatrix} -\frac{4}{7}, -\frac{2}{7} \end{bmatrix} x_1 + x_2 = \begin{bmatrix} \frac{23}{7}, \frac{25}{7} \end{bmatrix}$$

and the initial box [-2.5,2.5]×[1.5,4.5].

- a) Use the Interval Gauss-Seidel method to solve the system without precontitioning. (page 38)
- b) Compute an adequate preconditioner *P*. (see page 39)
- c) Obtain an equivalent system by applying de above preconditioner *P*. (see page 40)
- d) Use the Interval Gauss-Seidel method to solve this new the system. (see page 41)

3. Nonlinear Systems of Equations (Pruning)

Apply the multivariate interval Newton method to solve the following systems:

- a) $x_1^2 + x_1 x_2 = 10$ with the initial box [1.75,2.25]×[2.75,3.75]. (see page 45) $x_2 + 3x_1 x_2^2 = 57$
- b) $x^2 = 5 y^2$ with the initial box [1,3]×[1,3]. $y + 1 = x^2$

4. Nonlinear Systems of Equations (Solving)

Compute enclosures for all solutions of the following system within the box $[0,4] \times [0,4]$:

 $x^2 + y^2 = 10$ $0.4y - 0.1x^2 = 0$