Continuous Constraint Programming

Intervals, Interval Arithmetic and Interval Functions. (see Lecture2.pdf)

1. Test the Interval Arithmetic Library

Consider the intervals $I1$=[0,1], $I2$=[2,3] and $I3$=[−2,−1] and compute:

a) The left and right bounds of the intervals. (see page 10)
b) The center and width of the intervals. (see page 10)
c) $I1+I2$, $I2-I3$, $I1\times I2$, $I2/I3$ and $I2/I1$. (see page 13)
d) $I1\cap I2$, $I1\uplus I2$, $(I2\uplus I3)\cap(2I1)$ and $(I3)^3$. (see page 9)
e) $I1\times(I2+I3)$ and $I1\times I2+I1\times I3$. (see page 15)
f) Let $I1$=[0.5,1], $I2$=[2,2.5] and $I3$=[−2,−1] and compute again e). (see page 15)

2. Interval Functions

Consider the interval expressions $X1-X1^2$, $X1\times(1-X1)$ and $0.25-(X1-0.5)^2$.

a) Evaluate each expression with $X1$=[0.5,2]. (see page 29)
b) For each expression, evaluate with $X1$=[0.5,1.25] and with $X1$=[1.25,2], and compute the union hull of the results. (see page 30)

3. Interval Extensions

Consider the univariate polynomial function expressed in the standard form as: $f(x) = x^3 - x^2 - x$

a) Express this function in the Horner form. (see page 33)
b) Express this function in the Factored form. (see page 33)
c) Compute the enclosure for the range of the function in [-1,1] with each of the 3 forms. Which one is tighter?
d) Use the form that produced the tighter enclosure in c) and compute a smaller enclosure based on the subdivision of the previous interval into 4 intervals of width 0.5. (see page 34)
e) Define a function that based on the monotonicity of f computes a sharp enclosure of the range of the function for any interval [a,b]. (see page 35)
f) Define a function that computes an enclosure obtained by the mean value extension of f over any interval [a,b] centered at the midpoint. (see page 36)

4. Sharp enclosure of the range of a real function

Implement a function that given a generic real function f, an arbitrary interval [a,b] and a precision ε, computes a sharp enclosure of the range of f in [a,b] with the following procedure:

a) Computes a sequence of intervals that contain all the zeros of the derivative of f. All the intervals must have the width smaller that the given precision ε.
b) Computes the enclosure of the range of f in [a,b] based on the obtained sequence of intervals.