

# Continuous Constraint Programming

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*The C++ library for constraint processing over real numbers. First steps.*

## 1. The Interval-Based EXplorer library (IBEX): <http://www.ibex-lib.org/>

- a) Download the latest release (<http://www.ibex-lib.org/download>) and install in your working environment (<http://www.ibex-lib.org/doc/install.html>).
- b) Test with a simple problem (<http://www.ibex-lib.org/doc/install.html#compiling-a-test-program>).
- c) Install the graphical tool: Vibes (<https://github.com/ENSTABretagneRobotics/VIBES>). Use the pre-built binaries of the VIBes viewer provided for Windows, MacOS and Linux platforms (<https://github.com/ENSTABretagneRobotics/VIBES/releases>).
- d) Test with the example1.cpp problem (extracted from the examples provided in the IBEX library).

## 2. Graphics, Boxes and Contractors

Consider the problem presented in the theoretical classes:

$$\begin{aligned}x &\in [-2, 2] & y &\in [-2, 10] \\ y &= x^2 & y &\geq 2x + 4\end{aligned}$$

Use program example2.cpp to:

- a) Draw the graphics for  $y = x^2$  and  $y = 2x + 4$ .
- b) Draw the initial box.
- c) Draw the box resulting from contracting the initial box only wrt the constraint  $y = x^2$ .
- d) Draw the box resulting from contracting the initial box only wrt the constraint  $y \geq 2x + 4$ .
- e) Draw the box resulting from contracting the initial box by propagating the contraction of both constraints.
- f) Draw the two boxes resulting from the application of the propagation contractor to both boxes obtained from splitting the largest domain of the previous box.

## 3. System Solver and Global Optimizer

Consider the problem presented in the theoretical classes:

$$\begin{aligned}x &\in [-\pi, \pi] & x &\in [-\pi, \pi] \\ x^2 y + x y^2 &\leq 0.5\end{aligned}$$

Use program example3.cpp to:

- a) Call the system solver `IbexSolve` to compute a cover of the solution space from the specification in file `satisfaction_problem.txt`.
- b) Call the global optimizer `IbexOpt` to search for a solution that maximizes  $x + y$ . The respective minimization problem is specified in file `optimization_problem.txt`.

## 4. The Problem of the Seismic Epicentre

Consider the problem where the goal is to estimate the epicentre coordinates of a seismic event. The seismic waves produced have been recorded at a network of six seismic stations at different arrival times. The following table presents their coordinates and the observed arrival times.

$(x_i, y_i)$	(3 km, 15 km)	(3 km, 16 km)	(4 km, 15 km)	(4 km, 16 km)	(5 km, 15 km)	(5 km, 16 km)
$t_i$	3.12 s	3.26 s	2.98 s	3.12 s	2.84 s	2.98 s

It is assumed that: seismic waves travel at a constant velocity of  $v = 5 \text{ km/s}$ ; experimental uncertainties on the arrival times are independent and can be modelled using a Gaussian probability density with a standard deviation  $\sigma = 0.1 \text{ s}$ .

- Compute a cover of the possible epicentre coordinates of the seismic event assuming that the observation errors cannot exceed  $3\sigma$ .
- Compute the maximum likelihood point, that is, the epicentre coordinates that minimizes the sum of the squared errors.
- Show graphically the results obtained.