Constraint Programming

2020/2021 – Mini-Test #2

Monday, 11 January, 9:00 h in 204-Ed.II

Duration: 1.5 h (open book)

1. Interval Arithmetic

Consider the univariate polynomial function expressed in the standard form as:

$$f(x) = x^3 - 7x - 6$$

1.1. Define the mean value extension of f over the interval $[-3/2, -1/2]$ centered at the midpoint.

1.2. Let $I = [-1 - w, -1 + w]$ with $w = 1/2$. Compute enclosures for the range of $f(I)$ with:
   a. the standard form;
   b. the centered form defined in 1.1

1.3. Prove that for any positive $w \leq 1/2$ the enclosure for the range of $f([-1 - w, -1 + w])$ obtained with the centered form is sharper that the obtained with the standard form.

1.4. Define an algorithm that based on the monotonicity of f computes a sharp enclosure of the range of the function for any interval $[a,b]$.

2. Interval Newton

Consider the polynomial of the previous question: $f(x) = x^3 - 7x - 6$

2.1. Define the interval Newton function for the polynomial.

2.2. Use the interval Newton method to compute an interval enclosure of the smallest root of the polynomial within $[-3,0]$. The enclosure must be certified (proved that contains a root) and sharp (width cannot exceed 0.05).

3. Constraint Propagation

Consider the constraint $yx^2 + xy^2 = 0.75$ and a box $B = [-1,1] \times [-1,1]$

3.1. Is the constraint box-consistent in box B?

3.2. Is the constraint hull-consistent in box B?

3.3. Compute the box B' obtained by applying HC4-revise on the constraint with the initial box B.