Constraint Programming

2017/2018 – Mini-Test #2

Thursday, 7 December, 11:00 h in Lab 114-II

Duration: 1.5 h (open book)

1. Interval Arithmetic

Consider the univariate polynomial function expressed in the standard form as: $f(x) = -4x^3 - x^2 + 2x$

1.1. Express this function in 2 equivalent forms:
   a. Horner form $a + x(b + x(c + dx))$
   b. Factored form $(x - r_1)(x - r_2)(x - r_3)$

1.2. Compute the natural interval evaluation for each of the 3 forms for *I*=[0,1].

1.3. For Horner form and the Factored form choose an interval for which the natural interval evaluation computes the exact bounds of the function.

1.4. Define an algorithm that computes the exact bounds of the function for any *I*=[*a*,*b*].

2. Interval Newton

Consider the polynomial of the previous question: $f(x) = -4x^3 - x^2 + 2x$

2.1. Define the interval Newton function for the polynomial.

2.2. Use the interval Newton method to prove that the polynomial has no roots in [0.75,1].

2.3. Can you prove with only 1 iteration of the interval Newton method that the polynomial has a root in [0.5,0.75]? Justify.

3. Constraint Propagation

Consider the following Continuous Constraint Satisfaction Problem:

variables: $x \in [0,1]$

$y \in [0,1]$

constraints: $21x - 18xy = 5$

$15x + 12y = 14$

3.1. Define a set of narrowing functions able to enforce hull-consistency on the constraints of the CCSP (for any box $B \subseteq [0,1]^2$).

3.2. Starting with the original domains box $B \subseteq [0,1]^2$, apply the above narrowing functions up to a fixed-point. What is the box obtained?

3.3. Show the results that would be obtained during the execution of a branch-and-prune algorithm with the pruning step based on the above narrowing functions and a branching step that splits the largest variable domain in its midpoint. Start with the original domains box $B \subseteq [0,1]^2$ and stop when the width of any variable domain is strictly smaller than 0.5.