Constraint Programming

2016/2017 – Exam

Friday, 13 January 2017, 10:30 h

Part II– Interval Constraints (1.5 h – open book)

1. Interval Arithmetic

Consider the polynomial function expressed in the standard form as: $f(x) = x^3 - 3x^2 + 2x$

1.1. Express this function in:

Horner's form: $a_0 + x(a_1 + x(a_2 + \cdots + x(a_{n-1} + a_n x)))$

Factored form: $(x - r_1)(x - r_2) \cdots (x - r_n)$

1.2. Compute the natural interval evaluation of each form for I=[-1,2].

1.3. For each form choose a non-degenerated interval for which the natural interval evaluation computes the exact bounds of the function.

2. Interval Newton

Consider the polynomial of the previous question: $f(x) = x^3 - 3x^2 + 2x$

2.1. Define the interval Newton function for the polynomial.

2.2. Define an algorithm that computes the exact bounds of the derivative of the polynomial function for any I=[a,b] with a>1.

2.3. Use the interval Newton method to prove that the polynomial has at least one root in $[\frac{7}{4}, \frac{7}{3}]$.

2.4. Can you prove with only 1 iteration of the interval Newton step that the polynomial has no roots in $[\frac{1}{4}, \frac{3}{4}]$? Justify.

3. Constraint Propagation

Consider the following Continuous Constraint Satisfaction Problem:

variables: $x \in [0,1]$

$y \in [0,1]$

constraints: $x^2 + 3xy = 1$

$x = y + \frac{1}{2}$

3.1. Define a set of narrowing functions able to enforce hull-consistency on the original constraints of the CCSP (for any box $B \subseteq [0,1]^2$).

3.2. Starting with the original domains box $B \subseteq [0,1]^2$, apply the above narrowing functions up to a fixed-point. What is the box obtained?

3.3. Show the results that would be obtained during the execution of a branch-and-prune algorithm with the pruning step based on the above narrowing functions and a branching step that splits the largest variable domain in its midpoint. Start with the original domains box $B \subseteq [0,1]^2$ and stop splitting when the width of any variable domain is strictly smaller than 0.5. Stop pruning when the width of any variable domain is strictly smaller than 0.1.